Noise Characteristics of a Turbulent Crosswind Jet

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The turbulent flow which results when a circular jet issues into a crosswind is a noise source which is encountered during take-off and landing of certain types of V/STOL aircraft. The acoustic intensity of the noise from this flow is calculated by extending the freejet model of Ribner[1,2] to those crosswind jets for which the ratio of jet velocity to crosswind velocity is between 6 and 10. The extended model accounts for the non-circular cross section, the shortened potential core region, and the flow characteristics in the initial mixing region of the crosswind jet. The result of this analysis is an expression for the directivity function of the acoustic intensity from a crosswind jet of a V/STOL aircraft in flight. The maximum acoustic intensity is calculated to be in a direction which is slightly forward of the sidelines direction.

Nomenclature

a_o	= speed of sound in quiescent medium with density of ρ_o
d	= diameter of jet aperture
$g(\tau)$	= temporal dependence of fluctuating-velocity correlation function
i	= acoustic intensity per unit volume of turbulence
I	= acoustic intensity
L	= length scale in the turbulence
M_a	= U_a/a_o, Mach number of the aircraft or crosswind
M_c	= U_c/a_o, Mach number of convection of turbulent eddies in the jet
M_j	= U_j/a_o, Mach number of the exit jet flow
r	= $\lvert\mathbf{x}\rvert$, distance from the observer to the jet
$R_{xx}(\lambda)$	= two-point double-velocity correlation function
s	= $\pi\lambda/L$
u	= fluctuating component of velocity in the jet
$\mathcal{U}$	= total velocity in the jet
U	= mean velocity in the jet
U_a	= velocity of the aircraft or the crosswind
U_c	= convection velocity of turbulent eddies in the jet
$U_{\mathrm{ȼ}}$	= maximum velocity in the jet
U_j	= exit velocity of the jet
$\mathbf{x}$	= position vector of the observer from the jet
$\mathbf{y}$	= position vector in the turbulence
α	= const
γ	= angle
θ	= angle
λ	= $\lvert\boldsymbol{\lambda}\rvert$
$\boldsymbol{\lambda}$	= separation vector in the turbulence
ρ_o	= density of the jet flow and crosswind
ξ	= coordinate along the crosswind jet
σ	= measure of the mean velocity profile in the jet
τ	= separation time
ω_f	= characteristic frequency in the turbulence

I. Introduction

AN important consideration to the potential use of V/STOL (vertical/short take-off and landing) aircraft around urban areas is the noise associated with this type of aircraft. It is therefore of interest to identify possible noise sources of these aircraft and to evaluate their acoustic characteristics. One such noise source which is encountered in the flight of V/STOL aircraft is the turbulent jet exhausting into a crosswind. This occurs in those aircraft which receive their augmented lift for take-off and landing from a high-speed flow directed towards the ground. When the aircraft is in transition between vertical or short take-off and conventional flight, the high-speed flow is exposed to a crosswind. It is important to assess the acoustic characteristics of the resulting flow in order to determine the effects of this noise source on surrounding areas. The objective of the analysis presented herein is to calculate the directivity function of the acoustic intensity from the crosswind jet. The analysis is an extension of the freejet model of Ribner[1,2] to a class of crosswind jets.

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II. Physical Description of the Crosswind Jet

The high-speed flow which exhausts from the actual lift propulsion unit of a V/STOL aircraft is modeled by a turbulent jet from a circular aperture. The jet exhausts with a subsonic velocity U_j into a uniform crosswind with velocity U_a and with the same density as the jet. Many of the properties of the resulting crosswind jet have been reported by several investigators.[3-6]

There are some general similarities between the crosswind jet and the corresponding jet which exhausts into quiescent surroundings (hereafter referred to as the freejet). Two flow regions in the crosswind jet can be identified. The first is the initial mixing region or potential core region during which the flow from the aperture becomes fully turbulent. The second region is characterized by the decay of the mean velocity and the intensity of the turbulence with distance along the jet.

There are several differences between the crosswind jet and the freejet which relate to the noise characteristics of the flow. In general the extent of these differences depends upon the ratio of the exit jet velocity U_j to the crosswind velocity U_a. One important feature of the crosswind jet is the distortion of the circular cross section of the jet. The data of Jordinson[4] show that for the velocity ratio U_j/U_a equal to six, the initially circular cross section of the flow is distorted within two jet diameters of the aperture (Fig. 1). The decelerating crosswind compresses the jet while the edges of the jet experience shear stresses by the crosswind. This eventually leads to the formation of counter-rotating vortices further downstream. These additional mechanisms for entrainment of external fluid into the jet result in the potential core of the crosswind jet being shorter than it is in the freejet. The data of Keffer and Baines[3] indicate that the length of the potential core in crosswind jets with velocity ratios of 6 and 8 is approximately 2.2 and 3.0 jet diameters, respectively, compared with approximately 6 jet diameters for the freejet

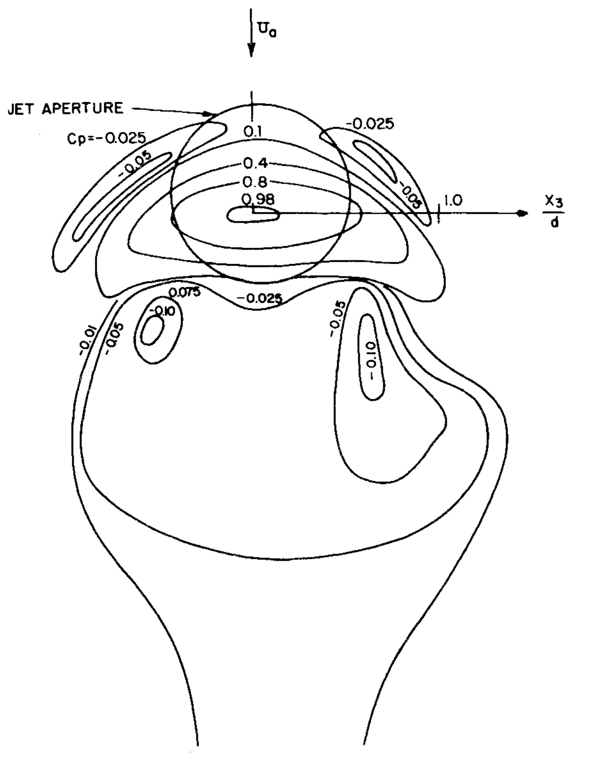

Fig. 1 Contours of pressure coefficient (C_p) for $U_j/U_a = 6.0$ at a position two jet diameters from exit (from Ref. 4); C_p = measured total pressure – wind total pressure/total pressure at aperture – wind total pressure.

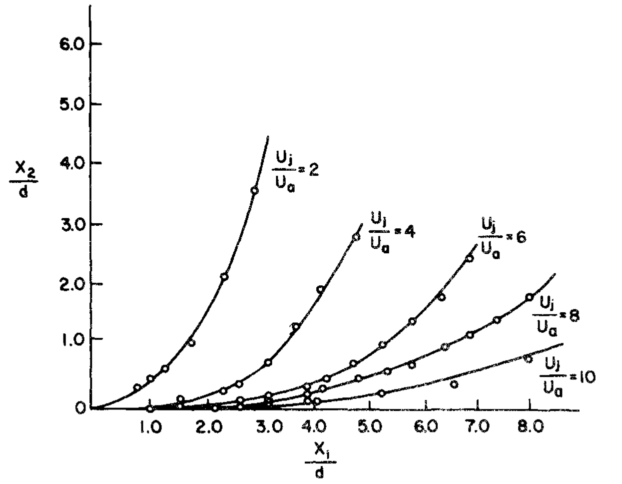

Fig. 3 Jet centerline as determined by measurements of maximum velocity for several values of U_j/U_a (from Ref. 3).

(Fig. 2). The additional entrainment mechanisms also result in a more rapid decay of the mean velocity in the second region of the crosswind jet. For a given exit jet velocity U_j, however, the intensity of the turbulence in the first several jet diameters is observed to be somewhat greater in the crosswind jet than in the freejet.[5] Another important feature is the deflection of the flow into the direction of the crosswind (Fig. 3). When the velocity ratio is 6 or greater, however, the centerline of the jet (as defined from the position of the maximum velocity in the cross section) is closely aligned with the exit direction in the initial mixing region of the jet. One final aspect of the crosswind jet to be noted is the region which is found downwind from the jet. The crosswind is partially deflected around the jet, and a wake region exists directly downwind (Fig. 1).

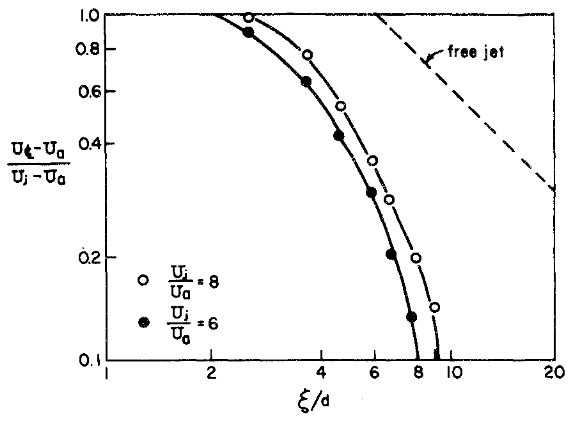

Fig. 2 Decay of jet centerline velocity compared to that of the freejet. Coordinate ξ is defined to be the direction of the jet centerline velocity (from Ref. 3).

III. Acoustic Analysis of the Flow

The analysis of the noise characteristics of the crosswind jet is based upon the aerodynamic noise theory of Lighthill[7] along with the contributions by Ffowcs Williams[8] and Ribner[1] to the convection factor. The turbulent crosswind jet is modeled following the work of Ribner[1,2] on the freejet.

The relationship between the acoustic intensity and the turbulence which generates the noise is given in terms of a correlation of the velocity in the flow. When the observer is far from the flowfield in terms both of the acoustic wavelength and of the physical dimensions of the region of turbulence, the relationship is given as follows

$$I(r) = \frac{\rho_o}{16\pi^2 a_o^5 r^2 [1 + (\mathbf{M}_a \cdot \mathbf{x}/r)]\{[1-(\mathbf{M}_c \cdot \mathbf{x}/r)^2] + \alpha^2 M_c^2\}^{5/2}} \times \frac{\partial^4}{\partial \tau^4} \int_\infty \int_\infty \overline{\mathscr{U}_x^2(\mathbf{y}, \tau)\mathscr{U}_x^2(\mathbf{y} + \boldsymbol{\lambda} t + \tau)}\, d^3\boldsymbol{\lambda}\, d^3\mathbf{y} \qquad (1)$$

where $r = |\mathbf{x}|$ is the distance of the observer from the jet, $a_o \mathbf{M}_a$ is the velocity of the aircraft, $a_o \mathbf{M}_c$ is the velocity at which the turbulent eddies are convected in the jet flow, α is an empirically determined parameter related to the characteristic frequency and length scale of the turbulence, $\mathscr{U}_x \equiv \mathscr{U} \cdot \mathbf{x}/r$ is the component of the total flow velocity in the direction of the observer, $\mathbf{y}$ is the position vector in the turbulent flow, $\boldsymbol{\lambda}$ is the separation vector between two points P and P′ in the turbulence (Fig. 4), τ is the

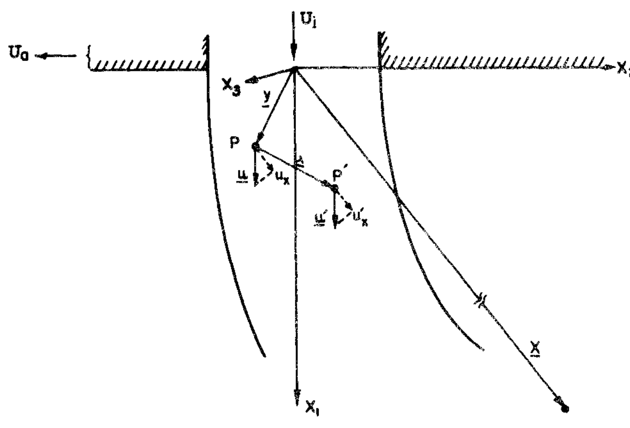

Fig. 4 Relationship among the quantities appearing in the equation for the acoustic intensity from the crosswind jet.

separation time which is related to $\boldsymbol{\lambda}$, and an overbar indicates the time-average. This equation for the acoustic intensity is identical with that given by Ribner[1] except for the addition of the convection factor $[1+(\mathbf{M}_a\cdot\mathbf{x}/r)]^{-1}$ from Ffowcs Williams[8] which accounts for the motion of the aircraft. In the following analysis the aircraft velocity is specified to be in the negative x_2-direction so that relative to the jet, the crosswind is directed in the positive x_2-direction. The intensity at a distance r from the jet is calculated by an integration of the velocity correlation over all separation distances at each point $\mathbf{y}$ and then over the entire region of turbulence.

The equation for the acoustic intensity is simplified following the procedure developed by Ribner.[1,2] The objective is to obtain an alternative expression for the acoustic intensity in which the flow characteristics of the crosswind jet can be more easily modeled. A brief description of the major steps in this procedure is presented in the discussion which follows.

The velocity correlation in Eq. (1) is expanded by expressing the total velocity component in the jet $\mathscr{U}_x$ as the sum of a mean velocity (U_x) and a fluctuating velocity (u_x). The turbulence in the jet flow is specified to be isotropic with a joint-probability distribution function which is Gaussian for the fluctuating velocity. Under these conditions the acoustic intensity which is generated by a unit volume of turbulence is expressed as follows:

$$i = \frac{\rho_o}{16\pi^2 a_o{}^5 r^2[1+(\mathbf{M}_a\cdot\mathbf{x}/r)]\{[1-(\mathbf{M}_c\cdot\mathbf{x}/r)^2]+\alpha^2 M_c{}^2\}^{5/2}}\times \frac{\partial^4}{\partial\tau^4}\int_\infty 2[(\overline{u_x u_x'})^2+2U_x U_x'\overline{u_x u_x'}]\,d^3\boldsymbol{\lambda} \tag{2}$$

where the unprimed and primed velocities denote evaluation at $(\mathbf{y},t)$ and $(\mathbf{y}+\boldsymbol{\lambda}, t+\tau)$, respectively. The terms which have no dependence on τ or which integrate to zero have been eliminated. The two terms which appear in the integrand give rise to contributions which are referred to as self-noise and shear-noise, respectively. The assumption is made that the second-order velocity correlation is separable into a spatial function $R_{xx}(\boldsymbol{\lambda})$ and a temporal function $g(\tau)$ such that $\overline{u_x u_x'}(\boldsymbol{\lambda},\tau) = R_{xx}(\boldsymbol{\lambda})g(\tau)$. As stated by Ribner,[2] the function $g(\tau)$ is assumed to have derivatives such that

$$\omega_f{}^4(g)^{iv} \equiv \left|\frac{\partial^4}{\partial\tau^4}g(\tau)\right|_{\tau=0} \quad\text{and}\quad \omega_f{}^4(g^2)^{iv} \equiv \left|\frac{\partial^4}{\partial\tau^4}g^2(\tau)\right|_{\tau=0}$$

The spatial dependence of the velocity correlation is taken to be the following function:

$$R_{xx}(\lambda) = \overline{u^2}[1-(\pi/L^2)(\lambda^2-\lambda_x{}^2)]\exp(-\pi\lambda^2/L^2) \tag{3}$$

where $\overline{u^2}$ is the square of the intensity of the turbulence, L is a characteristic length of the turbulence, and λ_x is the component of $\boldsymbol{\lambda}$ in the direction of the observer.

The self-noise portion of the acoustic intensity from a unit volume of turbulence at position $\mathbf{y}$ is calculated in terms of the intensity of the turbulence, the length scale, and the characteristic frequency at position $\mathbf{y}$ by integrating over all values of the separation vector $\boldsymbol{\lambda}$. This calculation is identical with that for the freejet, and an omni-directional pattern results with a magnitude which is given as follows:

$$i_{\text{self}} \equiv (\partial^4/\partial\tau^4)\int_\infty 2(\overline{u_x u_x'})^2\,d^3\boldsymbol{\lambda} = \omega_f{}^4[g^2(\tau)]^{iv}(\overline{u^2})^2L^3/2^{3/2} \tag{4}$$

The shear-noise portion of the acoustic intensity from a unit volume of turbulence is calculated by the following integral:

$$i_{\text{shear}} \equiv (\partial^4/\partial\tau^4)\int_\infty 4U_x U_x'\overline{u_x u_x'}\,d^3\boldsymbol{\lambda} = 4\omega_f{}^4[g(\tau)]^{iv}\int_\infty R_{xx}(\boldsymbol{\lambda})U_x U_x'(\lambda)\,d^3\boldsymbol{\lambda} \tag{5}$$

Since the function $R_{xx}(\boldsymbol{\lambda})$ is known, it remains to evaluate the mean-velocity correlation $U_x U_x'(\lambda)$ for the cross wind jet. It is noticed that the shear-noise integral is equal to zero when there are no spatial gradients in the mean velocity (i.e., when $U_x U_x'$ is a constant). In the circular freejet, the radial gradient of the axial velocity gives rise to the axially symmetric shear-noise pattern calculated by Ribner.[2] This symmetry is not present in the crosswind jet. As indicated in Fig. 1, the component of mean velocity directed along the centerline of the jet has a distribution which is approximately elliptical in the plane perpendicular to the centerline direction. Since the jet is deflected by the crosswind, the centerline of the jet is not in general aligned with the direction of the exit flow. Furthermore, in those portions of the edges of the jet which tend to be rolled-up by the crosswind, gradients of the mean velocity component in the crosswind direction are present. The mean velocity correlation is therefore not simply specified for the crosswind jet.

In view of the abovementioned considerations, the analysis is restricted first to the initial mixing region of the jet and second to a limited class of crosswind jets. It is expected that the region of large mean velocity gradients and turbulence intensity which is found in the initial mixing region of the crosswind jet is a substantial source of noise. Experimental determination of the source location in the freejet support this expectation when no crosswind is present.[9] With this restriction the mean velocity correlation need only be evaluated near the region of maximum shear in the initial mixing region of the crosswind jet. Furthermore, for the class of jets with velocity ratio U_j/U_a of 6 or greater, the data of Keffer[5] indicate that the centerline velocity in the initial mixing region is only slightly deflected by the crosswind. The mean velocity in the initial mixing region of the jet is therefore considered to be in the x_1-direction, and the mean velocity component in the direction of the observer is expressed as $U_x = U_1(x_1/r)$. It should be noted that the shear noise which results from the crosswind shear near the edges of the jet is proportional to $U_a{}^2$ while that which results from gradients of the centerline velocity component is proportional to $U_j{}^2$. The noise from the crosswind shear at the edges of the jet is therefore negligible for the restricted class of jets considered, and the dominant shear noise contribution is given by the following integral:

$$i_{\text{shear}} \cong 4\omega_f{}^4[g(\tau)]^{iv}\cos^2\theta\int_\infty U_1 U_1'(\boldsymbol{\lambda})R_{xx}(\boldsymbol{\lambda})\,d^3\boldsymbol{\lambda} \tag{6}$$

where θ is the angle between the observer position $\mathbf{x}$ and the x_1-axis (Fig. 5).

An expression for the correlation $U_1 U_1'(\boldsymbol{\lambda})$ is required which approximates the variation in mean velocity over the cross section of the jet. A reasonable choice for the curves of constant mean velocity is an elliptical distribution with the major and minor axes of the ellipse being aligned with the x_3- and x_2-axes, respectively. Thus, for the initial mixing region of the jet, the following variation of U_1 is assumed:

$$U_1 = U_j\exp[-(\pi/L^2)(\sigma_2\lambda_2{}^2+\sigma_3\lambda_3{}^2)]$$

where σ_2 and σ_3 are to be obtained from experimental

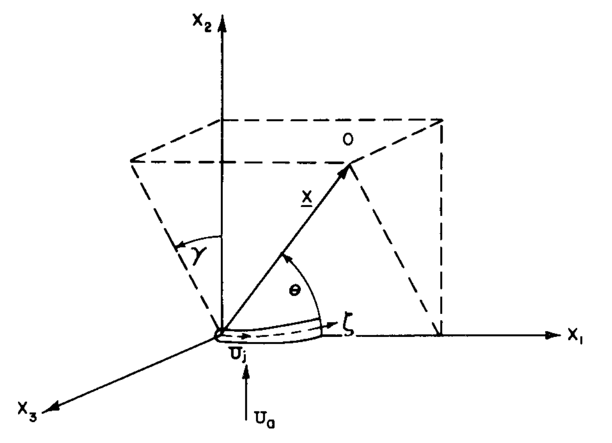

Fig. 5 Coordinate system for the crosswind jet.

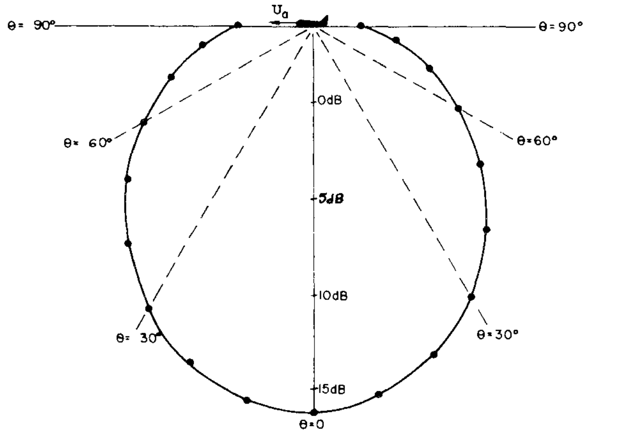

Fig. 6 Polar plot of the variation of the acoustic intensity in db as a function of angle θ. The velocity ratio is equal to six and $\gamma = 0°$, $M_j = 0.99$, $M_c = 0.54$, $M_a = 0.167$.

measurement. In the turbulent mixing region, the following approximation is then used for the correlation $U_1 U_1'(\boldsymbol{\lambda})$:

$$U_1 U_1'(\boldsymbol{\lambda}) = U_j^2 \exp[-(\pi/L^2)(\sigma_2 \lambda_2^2 + \sigma_3 \lambda_3^2)] \tag{7}$$

The distance λ_x which appears in the function R_{xx} must now be expressed in terms of components of $\boldsymbol{\lambda}$ in the x_1-, x_2-, and x_3-directions. As shown by Ribner,[2] this is done by rotating the coordinate system which has one axis in the **x**-direction through the angles θ and γ as shown on Fig. 5 to align this coordinate system with that which is defined by the directions of the jet exit velocity and the crosswind velocity. The result is the following:

$$R_{xx} = \overline{u^2}[1 - S^2 + S_1^2 \cos^2\theta + S_2^2 \cos^2\gamma \sin^2\theta + S_3^2 \sin^2\gamma \sin^2\theta + (\text{cross terms})] \exp(-S^2) \tag{8}$$

where $S_i \equiv (\pi)^{1/2}\lambda_i/L (i = 1, 2, 3)$ and $S = [(\pi)^{1/2}/L]\lambda$. In terms of the variable S, the integral for the shear-noise from a unit volume of turbulence is

$$i_{\text{shear}} = \frac{4\omega_f^4[g(\tau)]^{iv} U_j^2 \overline{u^2} L^3 \cos^2\theta}{\pi^{3/2}} \iiint_{\infty} [1 - S^2 + S_1^2 \cos^2\theta + S_2^2 \cos^2\gamma \sin^2\theta + S_3^2 \sin^2\gamma \sin^2\theta + (\text{cross terms})] \exp[-(S^2 + \sigma_2 S_2^2 + \sigma_3 S_3^2)]\, dS_1\, dS_2\, dS_3 \tag{9}$$

The cross terms provide no contribution, and the result of the integration is the following:

$$i_{\text{shear}} = \frac{4\omega_f^4[g(\tau)]^{iv} U_j^2 \overline{u^2} L^3 \cos^2\theta}{(1+\sigma_2)^{1/2}(1+\sigma_3)^{1/2}} \left[\frac{1 + \cos^2\theta}{2} - \frac{1 + \cos^2\theta + \sigma_2 \cos^2\gamma + \sigma_3 \sin^2\gamma + \cos^2\theta(\sigma_2 \sin^2\gamma + \sigma_3 \cos^2\gamma)}{2(1+\sigma_2)(1+\sigma_3)}\right] \tag{10}$$

Evaluation of this result for the shear noise requires experimental data on σ_2 and σ_3. Since this information is not readily available for the crosswind jet, the result is further simplified on the basis of the following qualitative argument. The jet expands relatively unimpeded in the x_3-direction; therefore, the value of σ_3 must be approximately equal to its counterpart in the freejet. From the values summarized by Ribner,[2] this parameter is in the range of 0.03 to 0.2. From the data of Jordinson[4] for a crosswind jet with velocity ratio equal to 6 (Fig. 1), the ratio of the major to minor axes of the cross section of the flow is approximately 3:1. With the assumed elliptical distribution of mean velocity in the cross section, the following ordering of the magnitude of the parameters must therefore exist: $\sigma_3 \ll \sigma_2$. This relationship is used to give the following simplified result for the shear-noise:

$$i_{\text{shear}} = \frac{2\omega_f^4[g(\tau)]^{iv} U_j^2 \overline{u^2} L^3 \sigma_2}{(1+\sigma_2)^{1/2}} \cos^2\theta(1 - \sin^2\theta\cos^2\gamma) \tag{11}$$

The preceding result is now combined with the expression for the self-noise to obtain the total intensity emitted from a unit volume of turbulence in the initial mixing region of the jet

$$i = \frac{\rho_o \omega_f^4 (\overline{u^2})^2 L^3 [g^2(\tau)]^{iv}}{2^{11/2}\pi^2 a_o^5 r^2 (1 + M_a \sin\theta\cos\gamma)[(1 - M_c \cos\theta)^2 + \alpha^2 M_c^2]^{5/2}} \times \left[1 + \left\{2^{5/2} \frac{[g(\tau)]^{iv}}{[g^2(\tau)]^{iv}} \frac{U_j^2}{\overline{u^2}} \frac{\sigma_2}{(1+\sigma_2)}\right\} \cos^2\theta(1 - \sin^2\theta\cos^2\gamma)\right] \tag{12}$$

The quantity in braces is similar to a corresponding quantity for the freejet which is estimated to have order of magnitude of unity.[2] In the present case, σ_2 is greater than its counterpart in the freejet. The normalized intensity of the turbulence $[\overline{u^2}]^{1/2}/U_j$, however, is observed to be greater in the crosswind jet than in the freejet. The value of the quantity in braces therefore is also estimated to have an order of magnitude equal to unity. The final result is then given as follows:

$$i(r, \theta, \gamma) = K(r) D(\theta, \gamma) \tag{13}$$

where

$$K(r) \equiv \frac{\rho_o \omega_f^4 (\overline{u^2})^2 L^3 [g^2(\tau)]^{iv}}{2^{11/2}\pi^2 a_o^5 r^2} \tag{13a}$$

and

$$D(\theta, \gamma) \equiv \frac{1 + \cos^2\theta(1 - \sin^2\theta\cos^2\gamma)}{(1 + M_a \sin\theta\cos\gamma)[(1 - M_c\cos\theta)^2 + \alpha^2 M_c^2]^{5/2}} \tag{13b}$$

IV. Results

The expression $D(\theta, \gamma)$ is the directivity function which provides the angular variation of the acoustic intensity at a given distance from the jet. Figure 6 shows this function in decibels [i.e., $10\,\mathrm{Log}_{10} D(\theta, \gamma)$] in the plane which is defined by the direction of flight and the exit direction of the jet. For this plane, $\gamma = 0$ and

$$D(\theta, 0) = \frac{1 + \cos^2\theta}{(1 + M_a \sin\theta)[(1 - M_c \cos\theta)^2 + \alpha^2 M_c^2]^{5/2}} \tag{14}$$

This pattern is very similar to that for the freejet except for the modification by the flight convection factor $(1 + M_a \sin\theta)^{-1}$. For this example the speed of the aircraft is taken to be $M_a = 0.167$ while the exit velocity of the jet is close to sonic so that $U_j/U_a = 6$. The data of Davies et al.[10] for the freejet

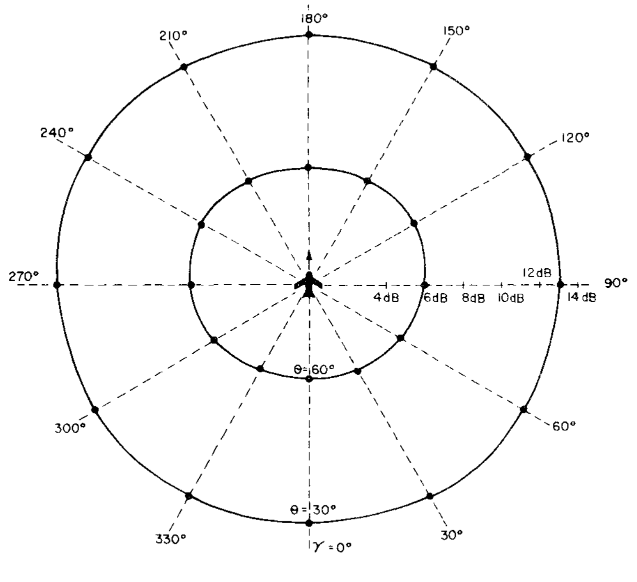

Fig. 7 Polar plot of the variation of the acoustic intensity in db as a function of angle γ. The dependence of the acoustic intensity on angle θ is given for cones of half-angle $\theta = 30°$ and $\theta = 60°$ below the aircraft. The velocity ratio is 6.0 and $M_j = 0.99$, $M_c = 0.54$, and $M_a = 0.167$.

indicates that $M_c = 0.54M_j$ and this value is used for the example calculations. Similarly the value of $\alpha = 0.55$ is used for the example. The effect of the movement of the aircraft is to increase the intensity in the direction in front of the aircraft and to decrease it behind the aircraft. For the case presented, this provides a maximum difference of 1.5 db directly ahead of and behind the aircraft. The model predicts a maximum intensity along the exit direction of the jet.

The dependence of the directivity function on angle γ is presented in Fig. 7 for the same jet and aircraft velocities as used for Fig. 6. The intensity for two angles θ and all values of γ give the variation in intensity around a cone of half-angle θ below the aircraft. The pattern is distorted from circular because of two effects. First, the convective effect of the motion of the aircraft increases the intensity in the direction in front of the aircraft and decreases it behind. Second, the elliptic variation in mean velocity over the cross section of the jet tends to give an increased signal at $\gamma = 90°$ and $\gamma = 270°$ compared with that at $\gamma = 0°$ and $\gamma = 180°$. The combination of these two effects produces a maximum acoustic intensity at angles $\gamma = 120°$ and $\gamma = 240°$. For $\theta = 60°$ this effect produces a difference of approximately 1.5 db between the maximum (at $\gamma = 120°$ and $240°$) and minimum (at $\gamma = 0°$) predicted values of the acoustic intensity. This prediction implies that the maximum jet noise from the lift propulsion units of a V/STOL aircraft under landing or take-off conditions is to be expected in a direction slightly forward of the sidelines direction.

V. Discussion

The result of this calculation is an expression for the directional behavior of the acoustic intensity emitted from a typical turbulent element in the noisy region of a crosswind jet with a velocity ratio of 6 or greater. It is expected that most of the initial mixing region of the crosswind jet emits sound in a manner similar to the specific region for which the model has been constructed. The effects of the crosswind which have been considered in this analysis such as the distortion of the cross section and the shortened potential core region become insignificant when the velocity ratio is greater than approximately ten. For such flows, the directivity function for the freejet[2] should give a reasonable prediction. The assumptions used in obtaining the directivity function are not reasonable for flows with a velocity ratio much less than 6.

The effects of signal refraction by mean velocity gradients in the jet flow and by the variation in crosswind velocity around the jet are not included in the calculation. As shown by Atvars et al.,[11] refraction by the mean velocity gradients in the freejet gives rise to a sharp decrease in intensity near the axis of the jet. A similar effect is expected to be present for the class of crosswind jet considered at positions near to $\theta = 0°$. Refraction of sound by the crosswind gradients near the jet are expected to be a weak effect for this class of crosswind jets. No ground reflections have been considered in this analysis. Furthermore, the results presented are not expected to provide a good comparison for positions directly downwind from the jet where the wake region exists.

While this theoretical prediction represents a reasonable extension of the model for a freejet, an experimental comparison with the predictions is more difficult to obtain than that for the freejet. With the development and flight of prototype V/STOL aircraft, fly-over data may provide such a comparison.

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